# The triple–alpha process and its anthropic significance

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**Abstract.** Through the triple–alpha process practically all of the carbon in our universe is synthesized as the ash of helium burning in red giants. The triple–alpha process proceeds trough the ground state of $^8$Be and though the $0^2_+$–state in $^{12}$C. We investigate the dependence of $0^2_+$–state and the production of carbon as a function of the strength of the underlying nucleon–nucleon interaction. This is performed by using the complex scaling method in a microscopic cluster model.

## 1 Introduction

The triple–alpha process occurring in helium burning of red giants is of special significance with respect to the anthropic principle [3, 2]. The anthropic principle deals with the question if our universe is tailor–made for the evolution of life. In other words, could life also have evolved in the universe, if the values of the fundamental constants or the initial conditions in the big bang were different. The reason for the relevance of the triple–alpha process with respect to the anthropic principle lies in the fact that one has to deal with physical quantities that lie in the realm of experimentally verifiable and theoretically calculable physics. This is for instance hardly the case for the rather uncertain and complicated science necessary for the description the big bang as well as for the creation and evolution of life on earth.

The formation of $^{12}$C through hydrogen burning is blocked by the absence of stable elements for the mass number $A = 5$ and $A = 8$. Öpik and Salpeter pointed out [8, 14] that the lifetime of $^8$Be is long enough, so that the $\alpha + \alpha \rightleftharpoons {}^8$Be reaction can produce macroscopic amounts of equilibrium $^8$Be in stars. Then, the unstable $^8$Be could capture an additional $\alpha$–particle to produce stable $^{12}$C. However, this so–called triple–alpha reaction has very low rate since the density of $^8$Be in the stellar plasma is very low because of its short lifetime of $10^{-16}$ s.

Hoyle argued [6] that in order to explain the measured abundance of carbon in the Universe, the triple–alpha reaction cannot produce enough carbon in a non–resonant way, but must proceed through a hypothetical resonance of $^{12}$C, thus strongly enhancing the cross section. Hoyle suggested that this resonance is a $J^\pi = 0^+$ state at about $\epsilon = 0.4$ MeV (throughout this paper $\epsilon$ denotes resonance energy in the center-of-mass frame relative to the three-alpha threshold, while $\Gamma$ denotes the full width). Subsequent experiments indeed found a $0^+$ resonance in $^{12}$C in the predicted energy region [6, 4]. It is the second $0^+$ state ($0_2^+$) in $^{12}$C. Its modern parameters $\epsilon = 0.3796\,$MeV and $\Gamma = 8.5 \times 10^{-6}\,$MeV [1] agree well with the old theoretical prediction.

In the following we discuss in Sect. 2 the used methods, i.e., the microscopic

three–cluster model, the effective nucleon–nucleon (NN) interactions, and the complex scaling method. In Sect. 3 we present the results for the triple–alpha reaction rates using different strengths of the NN–interaction. In Sect. 4 we discuss the astrophysical consequences of the obtained results.

## 2 The model

Our model is the a microscopic three–cluster ($\alpha + \alpha + \alpha$) resonating group model approach to the 12–nucleon system. Solving the 12–nucleon Schrödinger equation using a three–cluster trial function we get an equation for the intercluster relative wave function representing the three–body dynamics of the $^{12}$C states.

In order to avoid any possible model dependence of the conclusion we use three different effective NN–interactions: the Minnesota (MN) force designed to reproduce low–energy scattering data [11, 15], while the rather different Volkov 1 (V1) and 2 (V2) forces where obtained from fitting the bulk properties of s– and p–shell nuclei [16]. Each force contains an exchange mixture parameter, $u$ and $m$, respectively. The parameters were chosen to reproduce the experimental resonance energy of $\epsilon = 0.38\,\mathrm{MeV}$ for the $0_2^+$–state in $^{12}$C (MN: $u = 0.941$; V1: $m = 0.568$; V1: $m = 0.594$).

The three–body resonance energies for the $0_2^+$–state were determined by using the complex scaling method (CSM). It reduces the problem of asymptotically divergent resonant states to that of bound states, and can handle the Coulomb interaction without any problem.

A more detailed discussion of the model described in this section can be found in Ref. [9].

## 3 Reaction rates for the triple–alpha process

In this section we investigate the change of the reaction rate by varying the strength of all attractive and repulsive terms of the effective NN–potential through multiplication with a factor $p$. The consequences for triple–alpha reaction rate will be investigated, if this factor is changed by a very small amount of the order of 0.1 %.

The reaction rate for the triple–alpha process proceeding via the ground state of $^8$Be and the $0_2^+$–resonance in $^{12}$C is given by [12]

$$r_{3\alpha} = 3^{\frac{3}{2}} N_\alpha^3 \left( \frac{2\pi\hbar^2}{M_\alpha k_{\mathrm{B}} T} \right)^3 \frac{\omega\gamma}{\hbar} \exp\left( -\frac{\epsilon}{k_{\mathrm{B}} T} \right), \tag{1}$$

where $M_\alpha$ and $N_\alpha$ is the mass and the number density of the $\alpha$–particle, respectively. The temperature of the stellar plasma is given by $T$. The quantity $\epsilon$ denotes the difference in energy between the $0_2^+$–resonance in $^{12}$C and the $3\alpha$–particle threshold. The resonance strength $\omega\gamma$ is given by

$$\omega\gamma = \frac{\Gamma_\alpha \Gamma_{\mathrm{rad}}}{\Gamma_\alpha + \Gamma_{\mathrm{rad}}} \approx \Gamma_\gamma. \tag{2}$$

The approximation of the above expression for the decay widths of the $0_2^+$–resonance follows, because for the $\alpha$–width $\Gamma_\alpha$, radiation width $\Gamma_{\mathrm{rad}}$, the electromagnetic decay width $\Gamma_\gamma$ to the first excited state of $^{12}$C, and for the electron–positron pair emission decay width $\Gamma_{\mathrm{pair}}$ into the ground state of $^{12}$C the following approximations hold: (i) $\Gamma_\alpha \gg \Gamma_{\mathrm{rad}}$ and (ii) $\Gamma_{\mathrm{rad}} = \Gamma_\gamma + \Gamma_{\mathrm{pair}} \approx \Gamma_\gamma$.

Therefore, Eq. (1) can therefore approximated by:

$$r_{3\alpha} \approx 3^{\frac{3}{2}} N_\alpha^3 \left( \frac{2\pi\hbar^2}{M_\alpha k_{\rm B} T} \right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left( -\frac{\epsilon}{k_{\rm B} T} \right), \tag{3}$$

The two quantities in Eq. (3) that change its value by varying the effective NN–interaction is the energy of the $0_2^+$–resonance $\epsilon$ in $^{12}$C and its electromagnetic decay width $\Gamma_\gamma$. In Table 1 we show the change of the energy $\epsilon(p)$ of the $0_2^+$–resonance with respect to the $3\alpha$–threshold in $^{12}$C as a function of the multiplication of the strength factor $p$ for the three effective NN–interactions MN, V1 and V2. For no change we obtain again $\epsilon(1) = \epsilon$.

**Table 1.** Change of the energy $\epsilon$ of the $0_2^+$–resonance in $^{12}$C with respect to the $3\alpha$–threshold as a function of the strength factor $p$

| Effective NN–interaction | MN | V1 | V2 |
|---|---|---|---|
| $p$ | $\epsilon(p)$ [keV] | $\epsilon(p)$ [keV] | $\epsilon(p)$ [keV] |
| 1.002 | 327.5 | 337.5 | 343.7 |
| 1.001 | 353.7 | 358.7 | 361.7 |
| 1.000 | 379.6 | 379.6 | 379.6 |
| 0.999 | 405.2 | 400.3 | 397.2 |
| 0.998 | 430.5 | 420.8 | 414.6 |

It was found that the change of the reaction rate due to the the enhancement or reduction factor $f_p$ given below is larger by between two and three orders of magnitude than due to $\Gamma_\gamma$. Therefore, we neglected the dependence of the reaction rate on $\Gamma_\gamma$ by variations of the effective NN–interaction. The enhancement or reduction for the triple–alpha reaction rate is then given by

$$f_p = \frac{r_{3\alpha}(p)}{r_{3\alpha}} \approx \exp\left( \frac{\epsilon - \epsilon(p)}{k_{\rm B} T} \right). \tag{4}$$

In Table 2 the change of the triple–alpha reaction rate at a temperature of $10^8$ K given by the factor $f_p$ is shown as a function of the multiplication of the strength factor $p$ for the three effective NN–interactions MN, V1 and V2.

**Table 2.** Change of the triple–alpha reaction rate at a temperature of $10^8$ K as a function of the strength factor $p$

| Effective NN–interaction | MN | V1 | V2 |
|---|---|---|---|
| $p$ | $f_p$ | $f_p$ | $f_p$ |
| 1.002 | 422 | 132 | 64.4 |
| 1.001 | 20.2 | 11.4 | 7.9 |
| 1.000 | 1.0 | 1.0 | 1.0 |
| 0.999 | 0.05 | 0.09 | 0.13 |
| 0.998 | 0.003 | 0.008 | 0.02 |

Table 2 shows that the reaction rate $f_p$ at $10^8$ K is enhanced or reduced by the huge amount of about 4 orders of magnitude compared to the corresponding variations of the effective NN–interaction factor given by $p$. Furthermore, the model dependence due to the different used effective NN–interaction for $f_p$ is less than one

order of magnitude, and therefore much less than the before mentioned enhancement or reduction. Tables 1 and 2 also show at least for the considered small variations of the effective NN–interaction a linear scaling of $\epsilon$ and therefore an exponential scaling of $f_p$ with $p$.

## 4 Astrophysical consequences

The significance of low and intermediate and massive stars for the nucleosysnthesis of carbon is still unclear [5]. Some authors claim that AGB stars must be dominating in the production of carbon (e.g., [13]), whereas others favor the production of carbon in massive stars (e.g., [10]). In Ref. [7] the change of core helium burning in a massive star of $20\,M_\odot$ as well as shell helium burning in a AGB star of $5\,M_\odot$ was investigated. In this paper only hypothetical *ad hoc* shifts of the resonance energy of the $0_2^+$–state were investigated, whereas in this work we started by variations of the NN–interaction.

We can apply some of the results of Ref. [7] to our results. A lowering of the $0_2^+$ resonance energy by about 60 keV corresponding to a 0.2–0.4 % strengthening of the nucleon–nucleon interaction would lead to about a fourfold increase of the carbon production in a $20$–$M_\odot$ star. An increase of the $0_2^+$–state by about 60 keV corresponding to a 0.2–0.4 % weakening of the nucleon–nucleon interaction would lead to a decrease of roughly a factor two to three of the $^{12}$C–abundance in a $20\,M_\odot$ star. For a $5\,M_\odot$ star the situation is not so clear, since the change of carbon production the changes in the strength of the thermal pulses may compensate this effect. If the level is increased by about 650 keV corresponding to a about 2–4 % weakening of the NN–interaction (assuming a linear scaling of the resonance energy with the NN–interaction) then practically no more carbon could be produced in core and shell helium burning.

## 5 References

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